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Erratum: Eigenvectors of Backwardshift on a Deformed Hilbert Space

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After the publication of my paper (Das, 1998) I found a serious mistake in the proof of overcompleteness of coherent vectors. The correct version is as follows.

The coherent vectors are complete, in fact, overcomplete—they form a resolution of the identity

$$I = \frac{1}{2\pi} \int_{\alpha \in \mathbb{C}} d\mu(\alpha) \, |f(\alpha)\rangle\langle f(\alpha)| \tag{1}$$

where

$$d\mu(\alpha) = e_q(|\alpha|^2) e_q(-|\alpha|^2) \, d_q|\alpha|^2 \, d\theta \tag{2}$$

where $\alpha = re^{i\theta}$.

To prove this, we define the operator

$$|f(\alpha)\rangle\langle f(\alpha)|: \quad H_q \rightarrow H_q \tag{3}$$

by

$$|f(\alpha)\rangle\langle f(\alpha)| f(z) = (f(\alpha), f(z)) f(\alpha) \tag{4}$$

with $f(z) = \Sigma_0^\infty \, b_n z^n$. Now,

$$(f(\alpha), f(z)) = e_q(|\alpha|^2)^{-1/2} \sum_{n=0}^{\infty} [n]! \, \frac{\bar{\alpha}^n}{[n]!} b_n$$

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Then,

$$(f(\alpha), f(z))f(\alpha) = e_q(|\alpha|^2)^{-1} \sum_{m,n=0}^{\infty} \frac{\alpha^m}{\sqrt{[m]!}} \bar{\alpha}^n b_n f_m$$

Hence,

$$\begin{aligned}
\frac{1}{2\pi}\int_{\alpha\in\mathbb{C}} d\mu\, |f(\alpha)\rangle\langle f(\alpha)|f(z) &= \sum_{m,n=0}^{\infty} \frac{f_m}{\sqrt{[m]!}} b_n \frac{1}{2\pi}\int_0^{\infty} d_q r^2\, e_q^{-r^2}\, r^{m+n} \\
&\quad \times \int_0^{2\pi} d\theta\, e^{i(m-n)\theta} \\
&= \sum_{n=0}^{\infty} \frac{f_n}{\sqrt{[n]!}} b_n \int_0^{\infty} d_q r^2\, e_q^{-r^2}\, r^{2n} \\
&= \sum_{n=0}^{\infty} \frac{f_n}{\sqrt{[n]!}} b_n \int_0^{\infty} d_q x\, e_q^{-x}\, x^n \\
&= \sum_{n=0}^{\infty} \sqrt{[n]!}\, b_n f_n \\
&= f(z) \qquad (5)
\end{aligned}$$

where I have taken $x = r^2$ and utilized the fact that $\int_0^{\infty} d_q x\, e_q^{-x}\, x^n = [n]!$ (Gray and Nelson, 1990).

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